

Implementing inverse seesaw mechanism in $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ gauge models

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Abstract

Generating appropriate tiny neutrino masses via inverse seesaw mechanism within the framework of a particular $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ gauge model is the main outcome of this letter. It is achieved by simply adding three singlet exotic Majorana neutrinos to the usual ones included in the three lepton quadruplet representations. The theoretical device of treating gauge models with high symmetries is the general method by Cotăescu. It provides us with a unique free parameter (a) to be tuned in order to get a realistic mass spectrum for the gauge bosons and charged fermions in the model. The overall breaking scale can be set around 1-10 TeV so its phenomenology is quite testable at present facilities.

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1 Introduction

It is well-known that the Standard Model (SM) ([1] - [3]) - based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ undergoing a spontaneous symmetry breaking (SSB) in its electro-weak sector up to the universal $U(1)_{em}$ - is not a sufficient device, at least for some stringent issues in the particle physics today. When it comes to generating neutrino tiny masses [4, 5], the framework of the SM is lacking the needed ingredients, so one should call for some extra considerations which are less natural in the context. One of the ways out seems to be the enlargement of the gauge group of the theory as to include naturally among its fermion representations some right-handed neutrinos - mandatory elements for some plausible mass terms in the neutrino sector Yukawa Lagrangian density (Ld).

Among such possible extensions of the SM, the so called "3-3-1" and "3-4-1" classes of models- where the new electroweak gauge groups are $SU(3)_L \otimes U(1)_X$ [6] - [8] and $SU(4)_L \otimes U(1)_X$ [9] - [14] respectively - has meanwhile established themselves as much suitable candidates. Some systematic classifications [15] -

[19] of these SM-extensions have been done. In this paper we are concerned with a particular class of 3-4-1 models (namely the one that prohibits exotic electric charges) whose phenomenological analysis can be found in the literature, see Refs. [20] - [28]. The neutrino mass issue has been addressed [29] - [31] with viable results within the framework of such models.

Here we propose a slightly different approach from the canonical one, in the sense that we apply the prescriptions of the general method [32] of treating gauge models with high symmetries. Proposed initially by Cotăescu, it essentially consists of a general algebraical procedure in which electro-weak gauge models with high symmetries ($SU(N)_L \otimes U(1)_Y$) achieve their SSB in only one step up to the residual $U(1)_{em}$ by means of a special Higgs mechanism. The scalar sector is organized as a complex vector space where a real scalar field φ is introduced as the norm for the scalar product among scalar multiplets. It also ensures the orthogonality in the scalar vector space. Thus, the survival of some unwanted Goldstone bosons is avoided. This leads to a one-parameter mass spectrum, due to a restricting trace condition that has to hold throughout. The compatibility of this particular method with the canonical approach to 3-3-1 and 3-4-1 models in the literature was proved in some recent papers by the author [33] - [37]. In the case of the particular 3-3-1 models with right-handed neutrinos an appealing outcome [37] with only two physical massive Higgses with non-zero interactions finally emerged.

Once we established the framework in which the 3-4-1 gauge model of interest is treated, we exploit the realization of a kind of quasi-inverse seesaw mechanism [38] - [46] by simply adding 3 new exotic sterile Majorana singlets (N_R). Finally, the free parameter (let's call it a) is tuned in order to obtain the whole mass spectrum (including the neutrinos). An apparently unused up to now parameter η_0 in the general method proves itself here as the much needed "lepton number violating" coupling to achieve the Majorana mass terms for N_R in the neutrino sector.

The letter is organized as follows. It begins with a brief presentation of the model and its parametrization supplied by the general Cotăescu method (in Sec.2) and continues with the inverse seesaw mechanism worked out within this framework (Sec. 3) and the tuning of the parameters (Sec. 4) in order to obtain phenomenologically viable results for the neutrino masses. Some conclusions are sketched in the last section (Sec. 5).

2 The 3-4-1 gauge model

Let's start by presenting the anomaly-free particle content of the 3-4-1 gauge model of interest here. It comprises the following:

Lepton families

$$L_\alpha = \begin{pmatrix} e_\alpha \\ \nu_\alpha \\ N'_\alpha \\ N''_\alpha \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{4}^*, -1/2) \quad e_{\alpha R} \sim (\mathbf{1}, \mathbf{1}, -2) \quad (1)$$

with $\alpha = 1, 2, 3$.

Quark families

$$Q_{iL} = \begin{pmatrix} u_i \\ -d_i \\ D_i \\ D'_i \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}, -1/6) \quad Q_{3L} = \begin{pmatrix} d_3 \\ u_3 \\ U \\ U' \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}^*, 5/6) \quad (2)$$

$$d_{3R}, d_{iR}, D_{iR}, D'_{iR} \sim (\mathbf{3}, \mathbf{1}, -2/3) \quad (3)$$

$$u_{3R}, u_{iR}, U_R, U'_L \sim (\mathbf{3}, \mathbf{1}, 4/3) \quad (4)$$

with $i = 1, 2$.

The above representations (written in the usual notation) ensure the cancellation of all the axial anomalies by an interplay between families. This prevents the model from compromising its renormalizability by triangle diagrams. The capital letters are reserved for the exotic quarks (D_i , D'_i and U , U') in each family. They must be heavier than the ordinary quarks known from the SM in order to keep consistency with the low energy weak phenomenology.

To this fermion content one can add 3 Majorana exotic neutrinos $N_{\alpha R} \sim (\mathbf{1}, \mathbf{1}, 0)$ without the danger of spoiling the renormalizability. The advantage these 3 exotic neutrinos bring is that they can play a crucial role in realizing a particular sort of inverse seesaw mechanism[38] - [46].

Gauge bosons

The gauge bosons of the model are determined by the generators of the associated $su(4)$ Lie algebra, expressed by the usual Gell-Mann matrices $T_a = \lambda_a/2$. So, the Hermitian diagonal generators of the Cartan sub-algebra are in the fundamental representation:

$$D_1 = T_3 = \frac{1}{2} \text{Diag}(0, 1, -1, 0), \quad D_2 = T_8 = \frac{1}{2\sqrt{3}} \text{Diag}(0, 1, 1, -2) \\ D_3 = T_{15} = \frac{1}{2\sqrt{6}} \text{Diag}(-3, 1, 1, 1). \quad (5)$$

In order to discuss the phenomenology of this model, we employ the Cotăescu method of treating gauge models with high symmetries. For the sake of completeness we write down the electric charge operator in this very method. It naturally arises as: $Q^\rho = e \left[-\sqrt{\frac{3}{2}} T_{15}^\rho + \frac{1}{2} X^\rho \right]$ for each representation ρ . Hence, one can easily recover the above fermion representation (up to an unusual order in the quadruplets, that can be rearranged at any time).

In this basis the gauge fields are expressed by: A_μ^0 (corresponding to the Lie algebra of the group $U(1)_X$) and $A_\mu \in su(4)$, that can be put as

$$A_\mu = \frac{1}{2} \begin{pmatrix} D_\mu^1 & \sqrt{2}Y_\mu & \sqrt{2}X_\mu & \sqrt{2}X'_\mu \\ \sqrt{2}Y_\mu^* & D_\mu^2 & \sqrt{2}K_\mu & \sqrt{2}K'_\mu \\ \sqrt{2}X_\mu^* & \sqrt{2}K_\mu^* & D_\mu^3 & \sqrt{2}W_\mu \\ \sqrt{2}X'_\mu^* & \sqrt{2}K'_\mu^* & \sqrt{2}W_\mu^* & D_\mu^4 \end{pmatrix}, \quad (6)$$

with $D_\mu^1 = A_\mu^3 + A_\mu^8/\sqrt{3} + A_\mu^{15}/\sqrt{6}$, $D_\mu^2 = -A_\mu^3 + A_\mu^8/\sqrt{3} + A_\mu^{15}/\sqrt{6}$, $D_\mu^3 = -2A_\mu^8/\sqrt{3} + A_\mu^{15}/\sqrt{6}$, $D_\mu^4 = -3A_\mu^{15}/\sqrt{6}$ as diagonal bosons. These diagonal Hermitian generators will supply the neutral gauge bosons A_μ^{em} , Z_μ , Z'_μ and Z''_μ . Therefore, on the diagonal terms in eq.(6) a generalized Weinberg transformation (gWt) must be performed in order to consequently separate the massless electromagnetic field from the other three neutral massive fields. One of the two massive neutral fields is nothing but the Z^0 -boson of the SM. The details of the general procedure with gWt can be found in Ref. [32] and its concrete realization in the model of interest here in Refs. [19, 25] where the neutral currents for all Z_μ , Z'_μ and Z''_μ are completely determined and the boson mass spectrum as a function of the unique remaining free parameter (a) is calculated.

Scalar sector and spontaneous symmetry breaking

In the general method [32], the scalar sector of any $SU(N)_L \otimes U(1)_Y$ electro-weak gauge model must consist of n Higgs multiplets $\phi^{(1)}, \phi^{(2)}, \dots, \phi^{(n)}$ satisfying the orthogonal condition $\phi^{(i)} \cdot \phi^{(j)} = \varphi^2 \delta_{ij}$ in order to eliminate unwanted Goldstone bosons that could survive the SSB. Here $\varphi \sim (1, 1, 0)$ is a gauge-invariant real field acting as a norm in the scalar space and n is the dimension of the fundamental irreducible representation of the gauge group. The parameter matrix $\eta = (\eta_0, \eta_1, \eta_2, \dots, \eta_n)$ with the property $Tr \eta^2 = 1 - \eta_0^2$ is a key ingredient of the method: it is introduced in order to obtain a non-degenerate boson mass spectrum. Obviously, $\eta_0, \eta_i \in [0, 1)$. Then, the Higgs Ld reads:

$$\mathcal{L}_H = \frac{1}{2} \eta_0^2 \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2} \sum_{i=1}^n \eta_i^2 \left(D_\mu \phi^{(i)} \right)^+ \left(D^\mu \phi^{(i)} \right) - V(\phi^{(i)}) \quad (7)$$

where $D_\mu \phi^{(i)} = \partial_\mu \phi^{(i)} - i(gA_\mu + g'y^{(i)}A_\mu^0)\phi^{(i)}$ act as covariant derivatives of the model. g and g' are the coupling constants of the groups $SU(N)_L$ and $U(1)_X$ respectively. Real characters $y^{(i)}$ stand as a kind of hyper-charge of the new theory.

For the particular 3-4-1 model under consideration here the most general choice of parameters is given by the matrix

$$\eta^2 = (1 - \eta_0^2) \text{Diag} \left(\frac{1}{2}a - b, \frac{1}{2}a + b, c - a, 1 - c \right), \quad (8)$$

where, for the moment, a, b and c are arbitrary non-vanishing real parameters. Obviously, $\eta_0, c \in [0, 1)$, $a \in (0, c)$ and $b \in (-a, +a)$. It obviously meets the trace condition required by the general method for any $a, b \in [0, 1)$. After imposing the phenomenological condition $M_Z^2 = M_W^2 / \cos^2 \theta_W$ (confirmed at the SM level) the procedure of diagonalizing the neutral boson mass matrix [25] reduces to one the number of parameters, so that the parameter matrix reads finally

$$\eta^2 = (1 - \eta_0^2) \text{Diag} \left(\frac{a}{2}(1 - \tan^2 \theta_W), \frac{a}{2}(1 + \tan^2 \theta_W), \frac{1-a}{2}, \frac{1-a}{2} \right), \quad (9)$$

while the 4 scalar 4-plets of the Higgs sector are represented by $\phi^{(1)} \sim (\mathbf{1}, \mathbf{4}, -3/2)$ and $\phi^{(2)}, \phi^{(3)}, \phi^{(4)} \sim (\mathbf{1}, \mathbf{4}, 1/2)$.

With the following content in the scalar sector of the 3-4-1 model of interest here and based on the redefinition of the scalar quadruplets from the general method as $\eta_1 \phi^{(1)} \rightarrow \chi$, $\eta_2 \phi^{(2)} \rightarrow \rho$, $\eta_3 \phi^{(3)} \rightarrow \zeta$ and $\eta_4 \phi^{(4)} \rightarrow \xi$

$$\begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^- \\ \chi_4^- \end{pmatrix} \sim (\mathbf{1}, \mathbf{4}, -3/2) \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^0 \\ \rho_4^0 \end{pmatrix}, \begin{pmatrix} \zeta_1^+ \\ \zeta_2^0 \\ \zeta_3^0 \\ \zeta_4^0 \end{pmatrix}, \begin{pmatrix} \xi_1^+ \\ \xi_2^0 \\ \xi_3^0 \\ \xi_4^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{4}, 1/2), \dots \quad (10)$$

one can achieve via the SSB the following vacuum expectation values (VEV) in the unitary gauge:

$$\begin{pmatrix} \eta_1 \langle \varphi \rangle + H_\chi \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \eta_2 \langle \varphi \rangle + H_\rho \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \eta_3 \langle \varphi \rangle + H_\zeta \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ \eta_4 \langle \varphi \rangle + H_\xi \end{pmatrix}$$

or equivalently

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{a(1 - \tan^2 \theta_W)} \langle \varphi \rangle + H_\chi \\ 0 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sqrt{a(1 - \tan^2 \theta_W)} \langle \varphi \rangle + H_\rho \\ 0 \\ 0 \end{pmatrix},$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ \sqrt{(1-a)} \langle \varphi \rangle + H_\zeta \\ 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sqrt{(1-a)} \langle \varphi \rangle + H_\xi \end{pmatrix} \quad (11)$$

.with the overall VEV

$$\langle \varphi \rangle = \frac{\sqrt{\mu_1^2 \eta_1^2 + \mu_2^2 \eta_2^2 + \mu_3^2 \eta_3^2 + \mu_4^2 \eta_4^2}}{\sqrt{2(\lambda_1 \eta_1^4 + \lambda_2 \eta_2^4 + \lambda_3 \eta_3^4 + \lambda_4 \eta_4^4) + \lambda_5 \eta_1^2 \eta_2^2 + \dots + \lambda_{10} \eta_3^2 \eta_4^2}}$$

resulting from the minimum condition applied to the potential

$$\begin{aligned}
V = & -\mu_1^2 \chi^\dagger \chi - \mu_2^2 \rho^\dagger \rho - \mu_3^2 \zeta^\dagger \zeta - \mu_4^2 \xi^\dagger \xi \\
& + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\zeta^\dagger \zeta)^2 + \lambda_4 (\xi^\dagger \xi)^2 \\
& + \lambda_5 (\chi^\dagger \chi) (\rho^\dagger \rho) + \lambda_6 (\chi^\dagger \chi) (\zeta^\dagger \zeta) + \lambda_7 (\chi^\dagger \chi) (\xi^\dagger \xi) \\
& + \lambda_8 (\rho^\dagger \rho) (\zeta^\dagger \zeta) + \lambda_9 (\rho^\dagger \rho) (\xi^\dagger \xi) + \lambda_{10} (\zeta^\dagger \zeta) (\xi^\dagger \xi)
\end{aligned} \tag{12}$$

The above potential has the simplest form allowed by both the gauge symmetry and the restrictive orthogonal condition in the scalar sector. The scales it provides are, obviously, splitted when parameter tuning favors $a \rightarrow 0$, such as $v \simeq v' \sim \sqrt{a}$ (responsible for the SM phenomenology) and $V = V' \sim \sqrt{1-a}$ (responsible for the heavier degrees of freedom in the 3-4-1 model) respectively. A more detailed redefinition (with worked out consequences for the Higgs sector) can be observed in the case of the 3-3-1 model (see Ref.[37]), but for our purposes here it needs no further development.

3 Implementing seesaw mechanism

With the above ingredients one can construct the Yukawa Ld allowed by the gauge symmetry in the 3-4-1 model. It simply give rise to the following terms:

$$\begin{aligned}
-\mathcal{L}_Y^\nu = & h_\rho \overline{L} \rho^\dagger N_R + h_\eta \overline{L} \zeta^\dagger N_R + h_\xi \overline{L} \xi^\dagger N_R + \frac{1}{2} h_\varphi \overline{N_R^c} \eta_0 \varphi N_R \\
& + \frac{1}{2\Lambda} h^m \varepsilon^{ijk} (\overline{L})_i (L^c)_j \left(\rho_k^\dagger \zeta_m^\dagger + \zeta_k^\dagger \xi_m^\dagger + \xi_k^\dagger \rho_m^\dagger \right) + h.c.
\end{aligned} \tag{13}$$

where all h s are 3×3 complex Yukawa matrices, their lower index indicating the particular Higgs each one connects with.

The Yukawa Ld leads straightforwardly to the following neutrino mass terms:

$$\begin{aligned}
-\mathcal{L}_{mass}^\nu = & h_\rho \overline{\nu_L} N_R \langle \rho \rangle + h_\zeta \overline{N_L^c} N_R \langle \zeta \rangle + h_\xi \overline{N_L^c} N_R \langle \xi \rangle \\
& + \frac{1}{2} h_\varphi \overline{N_R^c} N_R \eta_0 \langle \varphi \rangle + \frac{1}{2\Lambda} (h' - h'^T) \overline{\nu_L} N_L^c (\langle \zeta \rangle \langle \xi \rangle + \dots)
\end{aligned} \tag{14}$$

Evidently, Λ is the cut-off scale of the model, up to which it remains renormalizable as an effective theory. The Yukawa terms allow one to construct the quasi-inverse seesaw mechanism by displaying them into the following 9×9 complex matrix:

$$M = \begin{pmatrix} 0 & \frac{h}{\Lambda} (1-a) & h_\rho \sqrt{a(1+\tan^2 \theta_W)} \\ \frac{h^T}{\Lambda} (1-a) & 0 & H \sqrt{1-a} \\ h_\rho^T \sqrt{a(1+\tan^2 \theta_W)} & H^T \sqrt{1-a} & \frac{1}{2} h_\varphi \eta_0 \end{pmatrix} \langle \varphi \rangle \quad (15)$$

where $h = h' - h'^T$. Since $\langle \zeta \rangle = \langle \xi \rangle$, one can construct the Dirac mass term $H \overline{N_L} N_R \sqrt{1-a} \langle \varphi \rangle$. The new state N_L can be taken either N'_L or N''_L , as both exhibit the same quantum numbers and couplings to neutral currents (see couplings' Table in Ref. [25]). Therefore one can safely consider that $N'_L = N''_L = N_L$.

Due to the non-zero h_ρ this matrix is slightly different from the traditional inverse seesaw mechanism [38] - [40], but its resulting effects - we prove in the following - are phenomenologically plausible. However, this kind of seesaw matrix appears in the literature, see for instance Refs. [41, 42]. This 9×9 complex matrix can be displayed as:

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix} \quad (16)$$

with $m_D = \begin{pmatrix} \frac{h}{\Lambda} (1-a) & h_\rho \sqrt{a(1+\tan^2 \theta_W)} \end{pmatrix}$ a 3×6 complex matrix and $M_N = \frac{1}{2} \begin{pmatrix} 0 & H \sqrt{2(1-a)} \\ H^T \sqrt{2(1-a)} & h_\varphi \eta_0 \end{pmatrix}$ a 6×6 complex matrix acting in the seesaw formula.

By diagonalizing the above matrix one gets the physical neutrino matrices as: $M(\nu_L) \simeq -m_D (M_N^{-1}) m_D^T$ and $M(\nu_R, N_R) \simeq M_N$ which yield:

$$M(\nu_L) \simeq -\frac{(1-a)\eta_0 \langle \varphi \rangle}{\Lambda^2} \left[h (H^{-1})^T (h_\varphi) (H^{-1}) h^T \right] + \frac{\sqrt{a(1-a)(1+\tan^2 \theta_W)} \langle \varphi \rangle}{\Lambda} \left[(h_\rho) (H^{-1}) h^T + h (H^{-1})^T (h_\rho)^T \right] \quad (17)$$

$$\begin{pmatrix} M(\nu_R) & 0 \\ 0 & M(N_R) \end{pmatrix} = \begin{pmatrix} H \sqrt{1-a} + \frac{1}{2} h_\varphi \eta_0 & 0 \\ 0 & -H \sqrt{1-a} + \frac{1}{2} h_\varphi \eta_0 \end{pmatrix} \langle \varphi \rangle \quad (18)$$

The first term in Eq.(17) is obviously much smaller than the second one due to $\sim 1/\Lambda^2$.

Now, for the sake of simplicity and to get a rapid estimation of the masses, one can suppose without loosing the generality, the proportionality

$$h_\rho = \alpha H \quad (19)$$

Consequently, one gets the left-handed neutrino mass matrix as the complex 3×3 matrix:

$$M(\nu_L) \simeq \alpha \frac{\sqrt{a(1-a)(1+\tan^2\theta_W)} \langle \varphi \rangle}{\Lambda} (h^T + h) \quad (20)$$

It is evident that it is a pure Majorana mass matrix since $M(\nu_L)^T = M(\nu_L)$ holds. Assuming a natural order of magnitude, say $h_\rho, h, h_\varphi \sim O(1)$, one can estimate the order of magnitude of the individual masses in this matrix as

$$Tr M(\nu_L) \simeq 6\alpha \frac{\sqrt{(1+\tan^2\theta_W)} \langle \varphi \rangle_{SM}}{\Lambda \sqrt{1-\eta_0^2}} \quad (21)$$

since $m(W) = \frac{1}{2}g \langle \varphi \rangle_{SM} = \frac{1}{2}g \sqrt{(1-\eta_0^2)a} \langle \varphi \rangle$ (see [25]).

The right-handed neutrinos acquire some degenerate masses

$$M(\nu_R) \simeq M(N_R) \simeq \frac{1}{2} h_\varphi \eta_0 \langle \varphi \rangle \quad (22)$$

if $H \sim O(10^{-x})$, $\eta_0 \sim O(10^{-y})$ and $x \gg y$. Otherwise, for $x = y$, the exotic right-handed N_R s could come out as very tiny or even massless, while for the usual right-handed neutrinos $M(\nu_R) \simeq H \langle \varphi \rangle \simeq \alpha^{-1} h_\rho \langle \varphi \rangle$.

4 Tuning the parameters

Now one can tune the parameters in this particular 3-4-1 model in order to get phenomenologically viable predictions. Obviously, both $a, \eta_0 \in (0,1)$. Since η_0 is the parameter responsible with the lepton number violation, one can keep it very small, say $\eta_0 \sim 10^{-6}$ (or even smaller) in order to safely consider that the global $U(1)_{leptonic}$ symmetry is very softly (quite negligible) violated by the Majorana coupling it introduces.

When comparing the boson mass spectrum in this model - obtained both by using the general Cotăescu method [32] and the SM calculations [1] - one gets a scales connection:

$$\sqrt{(1-\eta_0^2)a} = \frac{\langle \varphi \rangle_{SM}}{\langle \varphi \rangle} \quad (23)$$

It becomes obviously that η_0 has no part to play in the breaking scales splitting. The later is determined quite exclusively by a . If one takes $\langle \varphi \rangle_{SM} \simeq 246 \text{ GeV}$ and $\langle \varphi \rangle \simeq 1 - 10 \text{ TeV}$ then $a \simeq (0.0006, 0.06)$.

With these plausible settings the individual neutrino masses come out in the subsequent hierarchy:

$$\sum m(\nu_L) \simeq 10^{12} \left(\frac{\alpha}{\Lambda} \right) eV \quad (24)$$

$$\sum m(\nu_R) \simeq 10^{-7} \langle \varphi \rangle \quad (25)$$

where $(\frac{\alpha}{\Lambda})^2 (1-a) \eta_0 \langle \varphi \rangle$ - as the first term in Eq.(17) - is negligible under the restriction $(\frac{\alpha}{\Lambda})^2 \sim 10^{-24}$ (from Eq.(24)) and $\langle \varphi \rangle \sim 10^{12} \text{eV}$.

One can observe from Eqs. (24) - (25) in the case with massless exotic N_{RS} , the smaller the masses of the right-handed ($\nu_R = N_L^c$) neutrinos, the higher the cut-off of the effective theory in this particular 3-4-1 model. That means, if $m(\nu_R) \sim 10^{-3} \text{eV}$ then the effective theory is valid up to $\Lambda \sim 10^{15} \text{GeV}$ which is the GUT scale. However this can be kept valid up to such high energies even though the right-handed massive neutrinos lie at any level in the sub-TeV region at the expense of assuming six quasi-degenerate such right-handed neutrinos.

Furthermore, one can enforce some extra flavor symmetries in the lepton sector in order to dynamically get the appropriate PMNS mixing matrix. Some discrete groups, such as A_4 [47, 48], S_4 [49] or S_3 [50, 51] were employed in 3-3-1, and the procedure can be similarly applied in order to accomplish this task in the model of interest here.

5 Concluding remarks

We discussed in this brief letter the possible realization of a quasi-inverse seesaw mechanism in a particular 3-4-1 gauge model with "lepton number violating" exotic Majorana neutrinos added. The Cotăescu general method of treating gauge models with high-symmetries is employed and works as a suitable framework for such a purpose. It successfully provides us not only with the one-parameter mass spectrum but also with the lepton number violating terms needed for a plausible inverse seesaw mechanism, due to the possibility of coupling the scalar φ to exotic Majorana neutrinos. To the extent of our knowledge, in low energy models one finds no such terms to give masses to exotic neutrinos, so that some extra assumptions (usually from GUT theories) are invoked. These two characteristics single out our approach from other recent similar attempts (for instance, in 3-3-1 models see Refs. [52] - [58]). The details of the mixing in the neutrino sector are closely related to the entries in h , h_ρ and h_φ but this lies beyond the scope of this letter and will be presented elsewhere.

Such SM-extensions are appealing for they proves themselves able to recover all the results of the SM and in addition exhibit a lot of assets: they require precisely 3 fermion generations, their algebraic structures dictate the observed charge quantization, they can predict a testable Higgs phenomenology and, as we presented here, are utterly promising for neutrino phenomenology.

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